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Quark Mass Corrections to the Perturbative Thrust and its Effect on the determination of α_s

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Abstract

We consider the effects of quark masses to the perturbative thrust in e^+e^- annihilation. In particular we show that perturbative power corrections resulting from non-zero quark masses considerably alters the size of the non-perturbative power corrections and consequently, significantly changes the fitted value of α_s .

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One of the cleanest signatures of perturbative QCD comes from jet cross sections in e^+e^- annihilation. In such processes, it is possible to define infra-red safe event shape variables which can be calculated order by order in perturbative QCD and compared subsequently with experiment. However in order to carry out these comparisons, a method has to be evolved to parametrise non-perturbative effects which though expected to be small at present Q^2 values at LEP, actually turn out to be substantial ($\sim 25\%$) even at $Q \sim m_Z$. One of the reasons for this is that these non-perturbative effects are actually suppressed by a single power of Q rather than Q^2 . In addition, it is also possible that these power corrections could be comparable to $\mathcal{O}(\alpha_s^2)$ at present LEP energies.

In order to address these issues the Milan group of Dokshitzer *et al.* [1] drawing on the earlier work of Webber [2], Korchemsky and Sterman [3] and others, presented a systematic approach for handling power corrections using perturbation theory. Very briefly, they studied the consequences of assuming that α_s has a low energy effective form which does not grow at low scales but has an infra-red regular form. The moments of α_s are integrated only over the infra-red region. Various non-perturbative parameters are then parametrised and the form and magnitude of power corrections are determined.

However before one uses the approach of the Milan group in order to get a handle on power corrections and subsequently determine α_s by a fit to the data, it is important to isolate power corrections coming from a purely perturbative region. The Milan approach neglects the masses of all the quarks but instead uses a gluon mass as a ‘trigger’ to differentiate the perturbative from the non-perturbative region. We find however that the masses of the quarks, particularly the **c** and the **b** quarks, even at present LEP energies, can contribute significantly (of the order of about 25%). In fact, if we go beyond the top quark threshold (which is expected, perhaps in the future NLC) the perturbative contribution to power contributions due the top quark mass is even larger. We will have more to say on this later in the paper.

In this paper, we consider the example of one such event shape variable - the thrust - and show the significance of the effect of quark masses which need to be folded in before estimating the non-perturbative contribution to power corrections. We present explicit expressions to $\mathcal{O}(\alpha_s)$ of quark mass corrections expanded to $\mathcal{O}(m)$. We also show the effect of keeping the full mass contribution to $\mathcal{O}(\alpha_s)$ which unfortunately does not have a simple analytic form like the former and needs to be calculated numerically. Using these expressions we then fold in the power corrections of the Milan type and use this full expression to estimate both α_0 and α_s and compare it with estimates that exist in the literature without taking quark masses into account.

The first paper which calculated the effect of quark masses to $\mathcal{O}(\alpha_s)$ was published about 16 years ago by one of the authors [4]. For completeness, in what follows, we quote those results from that paper which we need for our analysis here. The thrust, as defined traditionally, is given by

$$T = 2 \frac{\max \sum_{ich} (p_i \cdot \hat{n})}{\sum_i |p_i|}, \quad (1)$$

where the denominator runs over all observed particles and the numerator runs over all particles in a hemisphere. \hat{n} is a unit vector chosen in a direction that maximises the numerator and defines the jet axis.

While this definition is appropriate for all massless particles, to include mass effects in the definition of the thrust, we modify the above definition slightly and write

$$T = 2 \frac{\max \sum_{ich} (p_i \cdot \hat{n})}{W}, \quad (2)$$

where $W^2 = s$. Of course the denominator equals $\sum_i |p_i|$ when all the particles are massless. This normalisation with the total energy is also what is used by the Milan group in their analysis though in their case the massive gluon eventually decays into massless quarks and gluons.

For a three particle final state, the thrust, as we define it, is given by

$$T = \max \left[(x_1^2 - \xi)^{1/2}, (x_2^2 - \xi)^{1/2}, x_3 \right], \quad (3)$$

where $x_i = 2E_i/W$, E_i being the energy of the i th particle in the final state in the c.m. frame and $\xi = 4m^2/W^2$, m being the mass of the quarks. Note that in the two-jet limit $T = T_0 \equiv \sqrt{1 - \xi}$.

The average value of the thrust is defined by

$$\langle T \rangle = \frac{\left[\int T \frac{d\sigma}{dT} dT \right]}{\left[\int \frac{d\sigma}{dT} dT \right]}. \quad (4)$$

The numerator of the above is given up to $\mathcal{O}(\alpha_s)$ and to $\mathcal{O}(\xi)$ by ($\sigma_0 = (4\pi\alpha_2/s)e_i^2$ is the total cross section for $e^+e^- \rightarrow q_i\bar{q}_i$) [4]

$$\begin{aligned} \frac{1}{\sigma_0} \int T \frac{d\sigma}{dT} &= 1 - \frac{\xi}{2} + \frac{4\alpha_s}{3\pi} \left\{ \frac{137}{16}\xi + \frac{5}{4}\xi \ln 2 - \frac{1}{2}\xi^{1/2} + \frac{7}{9} + \frac{1}{4}\xi \ln^2 \xi - \xi \ln 2 \ln \xi + \frac{\pi^2}{6} \right. \\ &\quad - \frac{\xi\pi^2}{6} - \frac{1}{8}\xi \ln \xi - \frac{1}{2}\xi \ln 3 \ln 2 - \frac{9}{2}\xi \ln 3 - \ln^2 3 + \frac{3}{8} \ln 3 \\ &\quad - \frac{1}{3}\xi \left[Li_2(1 - \xi^{1/2} + \xi/2) - Li_2\left(\frac{1}{3} + \frac{1}{2}\xi\right) \right] - 2Li_2\left(\frac{2}{3} - \frac{1}{2}\xi\right) \\ &\quad \left. + \xi Li_2\left(\frac{2}{3} - \frac{1}{2}\xi\right) - \frac{1}{2}\xi Li_2\left(\frac{1}{3} - \frac{1}{4}\xi\right) + \xi \ln^2 2 \right\}, \end{aligned} \quad (5)$$

where $Li_2(x)$ is the dilogarithm function. In the $\xi \rightarrow 0$ limit this gives, for the average thrust,

$$\langle T \rangle = 1 + \frac{4\alpha_s}{3\pi} \left[\frac{1}{36} + \frac{\pi^2}{6} - \ln^2 3 + \frac{3}{8} \ln 3 - 2Li_2\left(\frac{2}{3}\right) \right], \quad (6)$$

which works out to, for the perturbative thrust in the massless limit,

$$\langle 1 - T \rangle = 1.05 \frac{\alpha_s}{\pi}, \quad (7)$$

as quoted in numerous places in the literature.

Several points here are worthy of note. The leading term in the $O(m)$ expansion above is $\xi^{1/2}$. Thus the leading mass correction goes as $1/Q$. To the best of our knowledge, this fact was noticed for the first time in [4] and subsequently in [2] and [1] who have traced it to appear from the soft phase space boundary. We would like to stress that this $1/Q$ behaviour is a pure perturbative higher twist effect to the thrust and not related to any non-perturbative contribution. Thus, it seems clear, that the coefficient of $1/Q$ in the full expression for the thrust would include contributions both from the perturbative as well as the non-perturbative sectors. This aspect will become more quantitative, when we do our fits later.

The second point to note is a calculational one. Since, in the two jet limit, the thrust is equal to $T_0 = \sqrt{1 - \xi}$, in order to make the virtual contributions vanish we need to calculate, not as in the usual case $\langle 1 - T \rangle$, but $\langle T_0 - T \rangle$. It is then a trivial matter to add a term $\langle 1 - T_0 \rangle$ to obtain $\langle 1 - T \rangle$ to compare with experiment.

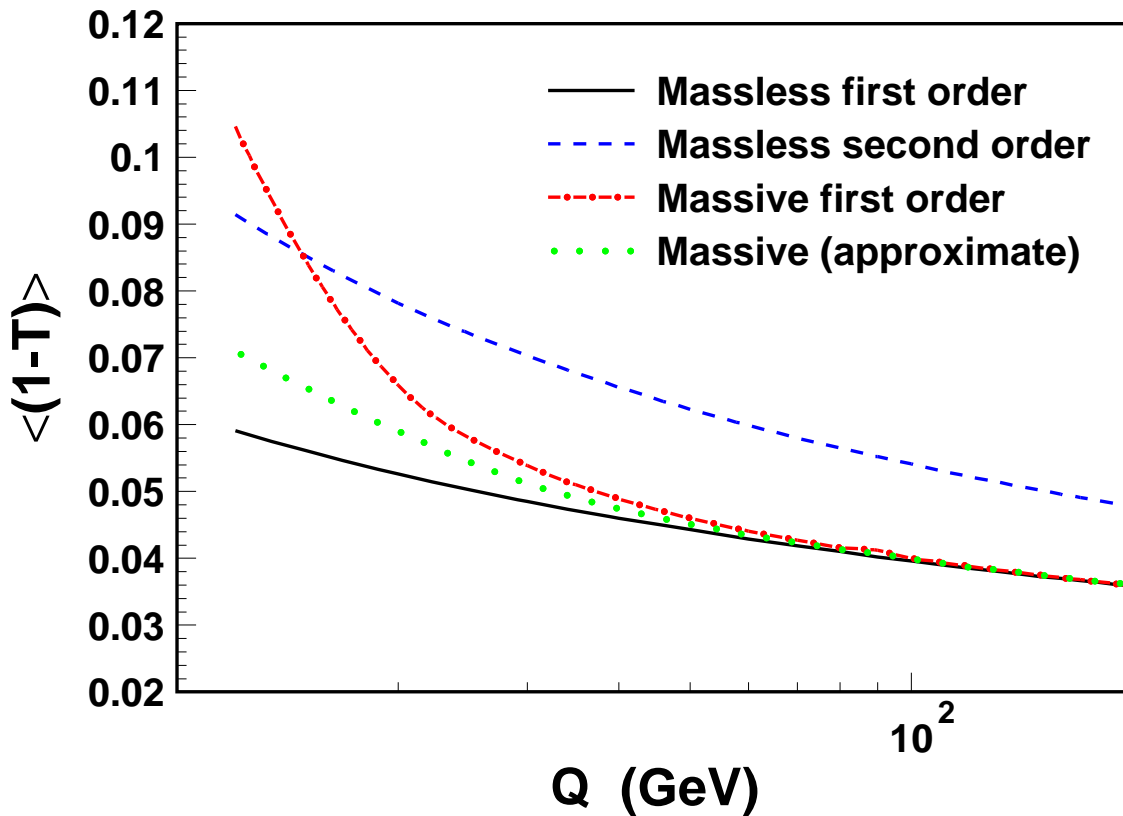


Figure 1: Effect of mass correction and higher order in the mean thrust value. The solid and the dashed lines are respectively the first and second order calculations of $\langle 1 - T \rangle$ without mass corrections. The dashed-dotted and dotted lines are complete and approximate ($\mathcal{O}(m)$) first order calculations with quark mass effects.

In order to compare with experiment, however, and to redo the fits for α_s and α_0 we have used not only the $\mathcal{O}(m)$ contribution above but also the full massive contribution to $\mathcal{O}(\alpha_s)$ albeit evaluated numerically. In addition we have also compared the $\mathcal{O}(\alpha_s^2)$ massless corrections to the $\mathcal{O}(\alpha_s)$ massive correction to try and estimate how much of the $1/Q$ corrections can be mimicked by higher orders in the coupling constant.

Figure 1 shows $\langle 1 - T \rangle$ as a function of centre of mass energy computed with $\alpha_s(m_Z) = 0.12$. As one sees from the curve the contribution of the second order terms are large over the entire energy region ($\sim 55\%$ at $Q = 12$ GeV going down to 33% at 200 GeV). On the other hand the effect of quark masses, evaluated only to first order in α_s , is even larger at low centre of mass energy ($\sim 76\%$ at $Q = 12$ GeV). This is clearly a $1/Q$ power law effect and hence dies off faster, becoming 2.5% at $Q \sim m_Z$ and negligible at 200 GeV. It is clear from the figure that one needs to take the full massive correction rather than the $\mathcal{O}(m)$ contribution, because it accounts only 60% (30%) of the mass correction at 20 GeV (12 GeV).

In order to compare the theoretical predictions with the measurements done at different centre of mass energies [5] at PETRA, PEP, TRISTAN, SLC and LEP, we add the non-perturbative contribution *a la* the Milan group [1] to the perturbative contribution. In this paper, we use only the $\mathcal{O}(\alpha_s)$ calculation of $\langle 1 - T \rangle$ and a more detailed comparison with a $\mathcal{O}(\alpha_s^2)$ calculation is under preparation [6]. We will have more to say on this later. The non-perturbative

contribution, as is well known, is given by an additive contribution $\langle 1 - T \rangle_{\text{pow}}$:

$$\langle 1 - T \rangle_{\text{pow}} = 2 \frac{4C_F}{\pi^2} \mathcal{M} \frac{\mu_I}{Q} \left[\alpha_0(\mu_I) - \alpha_s(Q) - \beta_0 \frac{\alpha_s^2(Q)}{2\pi} \left(\ln \frac{Q}{\mu_I} + \frac{K}{\beta_0} + 1 \right) \right] \quad (8)$$

where μ_I is an infra-red matching scale (taken as 2 GeV), $K = (67/18 - \pi^2/6) \cdot C_A - 5N_f/9$ and \mathcal{M} is the Milan factor (determined to be 1.49) [7].

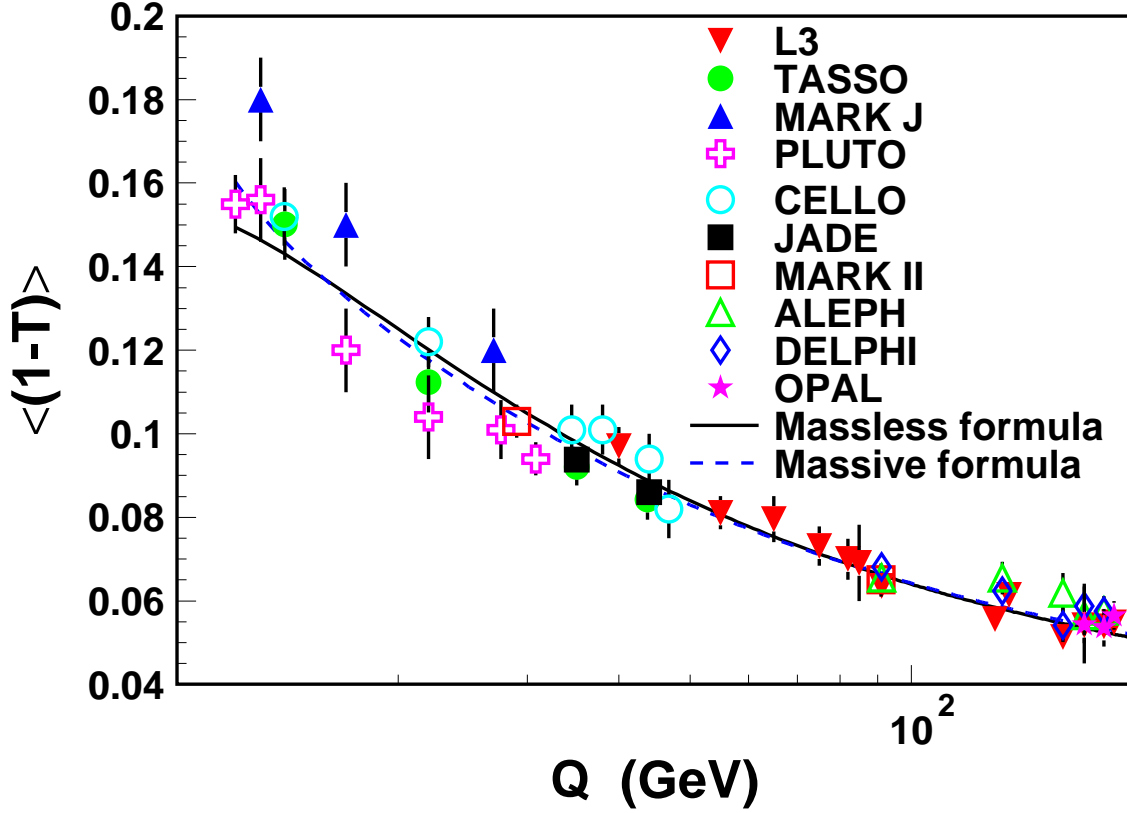


Figure 2: Results of the fit of $\langle 1 - T \rangle$ to the first order calculations with and without quark mass effects. The points indicate measured $\langle 1 - T \rangle$ values from different experiments and the solid and dashed lines are fits without and with mass corrections.

Figure 2 shows the experimental values of $\langle 1 - T \rangle$ together with the two fits which use respectively the massless and the massive forms (both to $\mathcal{O}(\alpha_s)$ for the perturbative contribution). These fits have been carried out with two free parameters $\alpha_s(m_Z)$ and α_0 . Both massless and massive formulation of the perturbative component give reasonable fits to the data with χ^2 of 90.2 and 69.2 respectively for 48 degrees of freedom corresponding to confidence levels of 0.22×10^{-3} and 0.24×10^{-1} . However, they do differ in the final values of $\alpha_s(m_Z)$ and α_0 as can be seen in Table 1. In these fits the scale parameter is chosen to be 1.0.

The errors quoted in the Table 1 are experimental errors obtained from the minimisation procedure. We can also estimate the theoretical uncertainties on these quantities by varying the scale parameter. If we vary the scale parameter between 0.5 and 2.0, we obtain uncertainties in α_s and α_0 to be ± 0.010 and ± 0.12 respectively. The value of $\alpha_s(m_Z)$, obtained from the fits, when quark mass effects are included or ignored, differ by 0.008 which is much larger than the experimental uncertainty of about .001 on the α_s value and comparable in fact to the theoretical uncertainty.

Fit Type	α_0	$\alpha_s(m_Z)$
Massless quarks	0.7931 ± 0.0066	0.1533 ± 0.0015
Massive quarks	0.7385 ± 0.0065	0.1612 ± 0.0014

Table 1: Results of the fits of the $\langle 1 - T \rangle$ distribution to perturbative and power law terms when the quark mass effects in the perturbative term is ignored or included

It is thus clear from the preceding analysis that an estimate of the power corrections due to the non-zero masses of the quarks is crucial in getting better and more realistic estimates on the strong coupling constant and indeed, in general, on power corrections. The next obvious step would be to calculate mass corrections to $\mathcal{O}(\alpha_s^2)$. Some results in this direction have been obtained by Nason and Oleari [8] which could be used to carry out a similar analysis to the one presented above. We are, at present, in the process of extending our analysis to second order in the strong coupling using the results of [8].

In various projected Linear Collider scenarios (like, for example, the NLC) energies upwards of 500 GeV are expected. In such a region, the effect of the top quark would be dramatic and significant. The combination of the large mass of the top quark and a charge squared of 4/9 implies that the usual massless expressions for the thrust would not work. We have estimated that the difference between choosing a massless formula for describing the thrust beyond the top quark threshold and using the (more appropriate) massive formula changes the value of the thrust by about a factor of 5 near the threshold. Most of this contribution comes, in fact, from the top quark mass. In the table below we give an estimate of the change that would occur between choosing all quarks massless and massive above the top quark threshold. It is obvious that the effect is spectacularly large, particularly near the threshold. Mass effects in the resummation of event shape variables are also expected to be significant and this is presently being studied.

Q (GeV)	α_s	$\langle 1 - T \rangle$ (massless)	$\langle 1 - T \rangle$ (Massive)
360	.0995	.0323	.1605
500	.0957	.0311	.0994
1000	.0888	.0289	.0435

Table 2: Difference between choosing massless and massive quarks above the top quark threshold

Thus, it is imperative that in order that reliable estimates be made of the thrust at these energies, we have available, calculations to higher orders in α_s of e^+e^- scattering with massive quarks in the final state. This would also give us a handle on the relative magnitudes of power corrections to the thrust to a particular order in α_s and the magnitude of the next order term in α_s [7,9]. For example NNLO effects might be capable of mimicking the $1/Q$ behavior. Mass effects in the resummation of event shape variables are also expected to be significant and this

is presently being studied.

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